Appendix A from A. Waananen et al., “Mating Opportunity Increases with Synchrony of Flowering among Years More than Synchrony within Years in a Nonmasting Perennial” (Am. Nat., vol. 192, no. 3, p. 379)

Mating Potential Equations

Whether an individual flowers on a day can be described by the binary function, \( F_{i,d} \). The function \( F_{i,d} \) is 1 if individual \( i \) is flowering on day \( d \) and 0 if not:

\[
F_{i,d} = \begin{cases} 1, \\ 0. \end{cases}
\]

Likewise, an individual flowering in each year, \( y \), can be described by the binary function \( F_{i,y} \), which equals 1 if individual \( i \) is flowering in \( y \) and 0 if not:

\[
F_{i,y} = \begin{cases} 1, \\ 0. \end{cases}
\]

The number of days (\( n_d \)) that an individual flowered per year is the sum of \( F_{i,d} \) over all days in a given year, \( \sum F_{i,d} \). The same sum over all days in the study period yields the number of days that an individual flowered of the entire study.

Turning to the annual temporal scale of resolution, the number of years (\( n_y \)) that an individual flowered is the sum of \( F_{i,y} \) over all years in the study period, \( \sum F_{i,y} \).

Daily mating potential (\( P_{\text{daily}} \)) is a count of all individuals flowering on that day. We calculate \( P_{\text{daily}} \) as the sum of \( F_{i,d} \) for all individuals on a given day \( d \):

\[
P_{\text{daily}} = \sum F_{i,d}.
\]

Similarly, we quantify mating potential at an annual scale of resolution (\( P_{\text{annual}} \)) as the sum of \( F_{i,y} \) in a given year, \( y \):

\[
P_{\text{annual}} = \sum F_{i,y}.
\]

We consider the mating opportunities of each day independent of those on any other day, even when the identities of individuals flowering are the same on multiple days. Thus, all mating opportunities in a year at a daily scale of resolution (\( T_{\text{daily,yr}} \) for total daily potential in a given year) can be described by the sum of \( P_{\text{daily}} \) over all days, \( d \), of a given year:

\[
T_{\text{daily,yr}} = \sum P_{d}.
\]

Similarly, all mating opportunities in the study period at an annual scale of resolution (\( T_{\text{annual,study}} \) for total annual potential in the study period) can be described as the sum of \( P_{\text{annual}} \) over all years of the study:

\[
T_{\text{annual,study}} = \sum P_{y}.
\]

Our goal was to measure the extent to which synchrony within and among years contributed to overall mating potential. To obtain a measure of overall mating potential, we quantified mating potential for individuals at a high-resolution temporal scale (daily) over the duration of the study period (11 years) in the following method. An individual’s mating potential on a given day is represented by the number of compatible mates participating in mating in the population on that day. For self-incompatible species, this can be represented by all individuals flowering in the population on that day except for the individual itself (i.e., \( P_{\text{daily}} = 1 \)). We calculate mating potential for individual \( i \) in a given year \( y \) as the average daily mating potential on the days that the individual flowered (mean daily mating potential, MDMP\(_{i,y}\)). Because this measure is not cumulative, it allows for comparison between individuals with different durations of flowering, such that individuals of varying total flowering time may be equally synchronous:
We calculate average MDMP, across all seasons that the individual flowered as an overall measure of mating potential, mean overall mating potential (MPo):

$$ MP_o = \frac{\sum_i MDMP_{i,y}}{n_y} $$

The previous measures of potential are absolute, in units of individual plants. However, comparing the extent to which within- and among-year synchrony contribute to overall mating opportunity requires (1) analogous measures at both temporal scales and (2) a measure of within-year synchrony that is independent of the variation in the number of individuals flowering in among years. Hereafter, we calculate measures of synchrony that are defined at two different temporal scales of resolution, but all range from 0 to 1.

To measure within-year synchrony for individual i (Wi), we calculate its mating potential as MDMP, and divide by the total daily mating potential in year y (Tdaily,year). By accounting for variation in the total number of individuals flowering in each year, this measure allows us to compare synchrony across years. Thus, within-year synchrony for individual i in year y can be represented as follows:

$$ W_{i,y} = \frac{MDMP_{i,y}}{T_{\text{daily,year}}} $$

We calculate mean Wi over all years that the individual flowered:

$$ W_i = \frac{\sum_y W_{i,y}}{n_y} $$

Our measure of among-year synchrony (Ai) was analogous to that of Wi, with potential measured at an annual rather than a daily scale of resolution. To account for variation in the total number of years that an individual flowered, we calculate mean annual mating potential (MAMPi,s) over the years that the individual flowered, thus allowing us to compare within-year synchrony between individuals with varying frequency of flowering:

$$ MAMP_{i,s} = \frac{\sum_y F_{i,y} \times (P_a - 1)}{n_y} $$

We then define Ai as the ratio of MAMPi,s to the total mating potential in the study period (Tannual,study):

$$ A_i = \frac{MAMP_{i,s}}{T_{\text{annual,study}}} $$
Appendix B from A. Waananen et al., “Mating Opportunity Increases with Synchrony of Flowering among Years More than Synchrony within Years in a Nonmasting Perennial” (Am. Nat., vol. 192, no. 3, p. 379)

Supplemental Methods and Figures

Nonparametric method for quantifying the relative importance of individual synchrony at two different temporal scales.

Methods

We assess the contribution of timing within and among years to individuals’ long-term mating potential by (1) creating a series of bootstrap null models for individuals that remove patterns of timing either within years, among years, or at both temporal scales; (2) calculating individual mating opportunity under empirical and null model scenarios; and (3) measuring the difference between empirical and each null model. We randomize individuals’ timing by reassigning either midpoint date of flowering (m) within a year, years flowering (Y), or both. Using two parameters, m and Y, to circumscribe timing allows us to manipulate each independently, enabling us to partition their relative contribution to overall mating potential. We investigate the four possible models of reassigning timing: reassigning m but not Y, Y but not m, both m and Y, and neither m nor Y. Unlike the models we use to evaluate population synchrony (see “Methods”), here we reassign the timing of only one individual at a time and calculate that individual’s mating potential given the actual population flowering pattern.

To account for seasonal variation, we recalibrate individuals’ midpoint date of flowering, m, relative to the mean flowering date of the year (i.e., the z-score; mz), providing a standardized measure of timing that translates across years. In models that maintain m, we use individuals’ mean mz, and in models that reassign m, we sample from the range of mz observed in the year to which timing is being assigned. The number of days that individuals flower (d) also varies among years. We use individuals’ median d to create our model flowering schedules. This parameterization is necessary in modeling traits, such as m and d, which may vary across an individual’s lifetime and be influenced by seasonal environmental variation. We compare the model reassigning neither m nor Y to observed mating potential to estimate error in the parameterization of m and d. For reassignments of Y, we sample from years of the study period, and we maintain the number of years that individuals flowered.

We iterated each model 500 times for every individual. The mean difference between individuals’ observed mating potential and the model scenario, or the change in cumulative number of potential mating interactions after reassigning timing, quantified the relationship between total mating potential and the temporal scale of reproductive timing that was reassigned in the model, m or Y. To measure the magnitude of the effect of within- and among-year timing, we calculated individuals absolute and mean percent change in individuals’ mating potential from observed to each model scenario (both within- and among-year timing reassigned, among-year timing reassigned, within-year timing reassigned, timing at neither scale reassigned). The mean change in mating potential under each model represents the effect of scale of timing that was reassigned in the model. We estimated the relative importance of within- and among-year timing for mating potential using a paired t-test to compare individuals’ mean absolute change in mating potential between models that reassigned only within- or among-year timing.

Results

Reassigning among-year timing resulted in greater mean absolute change in individuals’ mating opportunity than within-year timing (p < .001; fig. B1), indicating that among-season timing has a greater effect on long-term mating potential than within-season timing. Models reassigning among-season timing resulted in an average 17% change in mating potential from observed, compared to a 7% change in mating potential observed in models reassigning within-year timing. The mean percent change in mating opportunity of the model reassigning both scales of timing was 21%, while the model that reassigned neither scale of timing but calculated mating potential based on individuals’ mean
duration and median $m_t$ was 5%. Comparing mating opportunity from this model using parameterized values of $m$ and $d$ to the observed values indicated that our models slightly overestimated mating opportunity; using mean $m_t$ caused an increase of 1%, and median $n$ caused an increase of 2%.

![Figure B1](image-url)

**Figure B1:** Absolute differences between individuals’ observed mating potential and mating potential when among- and/or within-season timing are reassigned vary depending on the scale of timing that is reassigned. Among, within, both, and neither refer to the scale of timing that is reassigned in each simulation ($n = 500$) for all individuals ($n = 286$).
Appendix B from A. Waananen et al., Mating Opportunity Increases with Synchrony of Flowering Among Years More than Synchrony Within Years in a Nonmasting Perennial

![Graph showing observed and mid-domain effect (MDE) predicted flowering in each year of the study, 2005–2015.](image)

**Figure B2:** Observed and mid-domain effect (MDE) predicted (95\% confidence interval shaded) flowering in each year of the study, 2005–2015. On the Y-axis, “flowering plants” indicates the number of individuals flowering on each date. Lines represent mating potential at a daily scale of resolution. The value $S_{obs}$ refers to observed synchrony, while $S_{MDE}$ refers to mean synchrony of MDE simulations. The value $N_Y$ indicates the number of individuals that flowered in each year. Although the difference in the number of individuals flowering at the peak and the tails of the season between the MDE simulated and observed data may appear slight, this variation led to a strong pattern of greater-than-random synchrony in the observed data.
Figure B3: Histograms of synchrony values in mid-domain effect null models compared to observed synchrony in each year, indicated by the location of the red line in each panel (n = 10,000).
Figure B4: Proportion of individuals flowering in years with spring burns are indicated by red circles and in nonburn years by blue circles. The Y-axis represents the proportion of individuals flowering of all individuals planted in 1996 that were alive in the corresponding year. Evidence from natural populations suggests that fire stimulates flowering in Echinacea (S. Wagenius, unpublished manuscript). We observed that the lowest-flowering years (i.e., 2010, 2012, and 2014) were indeed summers that did not follow a spring burn. However, the relationship between burn years and high flowering was less clear, particularly in earlier years (i.e., 2007 was a nonburn year but was high flowering). The difference in the relationship between fire and flowering that we observed in this study compared to observations in a natural population could possibly be explained by age-specific variation in propensity to flower either in response to or in the absence of fire.